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A Combined integral transform asymptotic expansion method for the characterization of interface flaws through stimulated infrared thermography

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Abstract

This work is devoted to the nondestructive evaluation of materials using stimulated infrared thermography. The proposed approach provides simple analytical solutions to evaluate the lateral extent, and the thickness of a plane flaw in a three-dimensional (3D) heat transfer configuration. It is based on the application of a Laplace transform on the time variable *t*, then a double Fourier transform on the space variables *x* and *y*. Reduction of the models are obtained through an asymptotic expansion method. This mathematical formalism leads to the construction of explicit relationships that are very convenient for quantitative inversion. An experimental validation is performed on a calibrated carbon-epoxy laminate.

Keywords: stimulated infrared thermography, nondestructive evaluation, 3D heat transfer, quantitative inversion, composite material.

1. Introduction

Complex manufacturing processes of laminated composites increase the risk of flaw appearance whose consequences may be very crucial. In most cases, the control of their quality is needed to be nondestructive in order to allow inspection during different times of the parts life. In this quality investigation, methods based on heat transfer may be very effective. Pulsed infrared thermography is one of the approaches that may be used for that purpose. In this technique, the composite sample is irradiated by a uniform heat pulse on one face while the transient temperature either on the same face or on the opposite one is recorded using an infrared camera. The temperature difference between the pixel of interest on the infrared frame and a reference area considered sane on the same frame represents a signature of the subsurface flaw. It is this signal, usually called the contrast thermogram, which is commonly used to detect and quantify the flaw. In the current work, we propose a new modeling approach based on the use of the 3D thermal quadrupole method [1]. Three-dimensional thermal quadrupoles are obtained by applying to the real temperature field Laplace and Fourier transforms. Air delaminations in stratified composites are thermally characterized by a thermal contact resistance. When this resistance is small compared with the material whole resistance, a perturbation method [2] can be combined with the thermal quadrupole formalism to yield simple analytical solutions of the nondestructive experiment [3]. The perturbation procedure is based on an asymptotic expansion of the physical field versus a small parameter intervening in the model.

2. Formulation of the forward problem

The case of a rectangular *($L \times \ell$)* flat sample of thickness *e* that contains a resistive flaw of finite width *a* and finite length *b*, with a uniform contact resistance *R* on its whole area, is typical of a delamination in a composite material (Figure 1). In a typical pulsed infrared thermography test, one can assume that: the thermal excitation of the sample is a Dirac pulse characterized by a uniform absorbed energy *Q* (at time *t = 0*); the front face of the sample is adiabatic; and the sample temperature is zero before excitation.

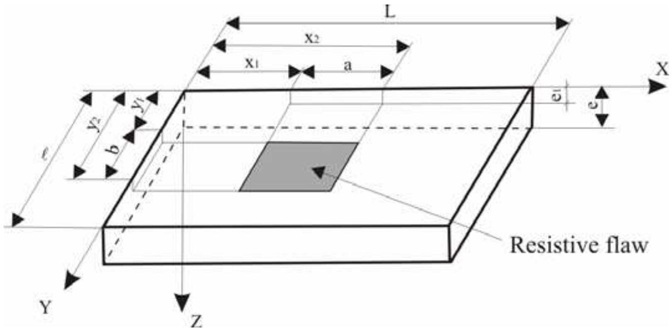

Figure 1. Geometric sketch of a limited extent flaw within an anisotropic material.

The Laplace transform $\tau(x, y, z, p)$ (where *p* is the Laplace variable) of the temperature *T(x, y, z, t)* in the sample is the solution of the following set of equations:

$$\frac{\partial^2\tau}{\partial z^2} + \frac{\lambda_x}{\lambda_z}\frac{\partial^2\tau}{\partial x^2} + \frac{\lambda_y}{\lambda_z}\frac{\partial^2\tau}{\partial y^2} - \frac{p}{a_z}\,\tau = 0 \qquad (1a)$$

$$x = 0, L \quad \rightarrow \quad \frac{\partial\tau}{\partial x} = 0 \qquad (1b)$$

$$y = 0, l \quad \rightarrow \quad \frac{\partial\tau}{\partial y} = 0 \qquad (1c)$$

$$z = 0 \quad \rightarrow \quad -\lambda_z\frac{\partial\tau}{\partial z} = Q \qquad (1d)$$

$$z = e_1 \quad \rightarrow \quad \frac{\partial\tau^{\sup}}{\partial z} = \frac{\partial\tau^{\inf}}{\partial z} \qquad (1e)$$

$$\tau^{sup} - \tau^{inf} = R\, s(x, y)\left[-\lambda_z\frac{\partial\tau}{\partial z}\right] \qquad (1f)$$

s(x, y) = 1 if (x, y) $\in$ *{[x_1, x_2] x [y_1, y_2]}* and *s(x, y) = 0* elsewhere

$$z = e \quad \rightarrow \quad \frac{\partial\tau}{\partial z} = 0 \qquad (1g)$$

This forward problem has already been solved elsewhere by the author [3], using 3D thermal quadrupoles and mathematical perturbations. Here, only pertinent results are recalled. After application of a double Fourier transform (along *x* and *y* directions) on the Laplace temperature distribution $\tau(x, y, z, p)$, it was shown that the thermal contrast on the rear face (non-irradiated face) could be written under the following form (for small *R*-values):

$$\Delta\theta = -R\frac{4}{\alpha\beta}K\frac{\sinh(u\,e_1)\sinh(\sqrt{p}\,(1-e_1))}{\sinh(\sqrt{p})\sinh(u)} \tag{2}$$

where $K = \sin\left(\alpha\frac{x_2-x_1}{2}\right)\cos\left(\alpha\frac{x_1+x_2}{2}\right)\sin\left(\beta\frac{y_2-y_1}{2}\right)\cos\left(\beta\frac{y_1+y_2}{2}\right)$

α and β are the Fourier variables, and $u = \left(p+\alpha^2+\beta^2\right)^{1/2}$

Return from the new (α, β, p) domain to the original (x, y, t) domain can be achieved numerically by using Fast Fourier Transform (FFT) and Stehfest [4] algorithms.

3. Explicit inversion of the lateral extent and thermal contact resistance

In this section, we show how the lateral extent of the flaw can be derived from the space averaging of the Laplace contrast distribution $\Delta\tau(x, y, z_s, p)$ when the depth of the flaw e_1 is already known. The space average of the Laplace contrast distribution is defined as:

$$\Delta\bar{\tau} = \frac{1}{L\ell}\int_0^L\int_0^\ell \Delta\tau(x,y,z_s,p)\,dx\,dy \quad ; \quad \text{with } z_s = 1 \text{ (rear face)} \tag{3}$$

Processing of Equation 2 for $\alpha = \beta = 0$ allows the calculation of $\Delta\bar{\tau}$ for the rear face:

$$\Delta\bar{\tau} = -R\frac{a\,b}{L\,\ell}\frac{\sinh(\sqrt{p}\,e_1)\sinh\left\lfloor\sqrt{p}(1-e_1)\right\rfloor}{\sinh^2(\sqrt{p})} \tag{4}$$

This shows that the average Laplace contrast (and therefore the averaged contrast $\Delta\bar{T}$ in the original space) is proportional to the product of the area *(a x b)* and the thermal resistance *R* of the flaw. To be able to estimate the lateral extent of the flaw, the thermal resistance *R* must be independently estimated. A one-dimensional (1D) inverse technique suitable for wide enough delaminations [5] could be used to estimate *R* above the center (x_c, y_c) of the defective area. If n_1 and n_2 denote the experimental Laplace contrasts on the rear face upon the center of the flaw, the thermal resistance is given by the following relationship:

$$R = \frac{n_1^2\sinh(\sqrt{p_1})\tanh(\sqrt{p_1})}{n_2\cosh(\sqrt{p_1}) - n_1} \tag{5}$$

where $n_i = \Delta\tau(x_c, y_c, 1, p_i)$, $i = 1, 2$ and $p_2 = 4\,p_1$

In the case of a flaw of small resistance *R*, application of Equations 4 and 5 allows the determination of the flaw extent if its depth e_1 is known is *a priori* known.

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4. Experimental validation

4.1. Measurement technique

The test sample is a square slab *60 mm x 60 mm* and *2-mm*-thick, made out of a 14-layer carbon-epoxy laminate. The material is orthotropic and its thermal properties are: $\lambda_z = 0.67\ W\,m^{-1}\,K^{-1}$, $\lambda_x = \lambda_y = 2.40\ W\,m^{-1}\,K^{-1}$, $\rho c = 1.62\ 10^{-6}\ J\,m^{-3}\,K^{-1}$. The delamination is simulated with a *10 mm x 10 mm* square artificial insert made out of two 25-μm-thick Teflon® films. These inserted films simulate a 3-μm-thick air delamination located at mid-depth in the sample ($e_1 = 0.5$). Heat pulse excitation is produced by an assembly of four flash tubes. Each flash tube is located on one side of a *10 cm x 10 cm* square frame. The duration of the photothermal excitation is less than *5 ms* for an incident energy *Q* of *3* to $4\ J\,cm^2$. The temperature change on the rear face of the sample after the flash heating is recorded with a Thermovision 782 SW Agema infrared camera connected to a computer system controller. The optical scanning gives a scan rate of 25 images per second and an image resolution of 128 pixels by 64 lines; the thermal resolution is around $0.08\ ^oC$.

4.2. Results

An instantaneous infrared frame of the rear face shows the flaw region on Figure 2. Application of Equation 5 with $p_1=1$ at the level of the defect center, leads to the following value of the normalized flaw thermal resistance $R = 0.185$. This value is almost three times higher than the nominal value corresponding to the Teflon® films. Previous experiments on similar calibrated samples showed that air layers of few micrometers trapped between the Teflon® films or between the films and the composite matrix are sufficient to justify this deviation.

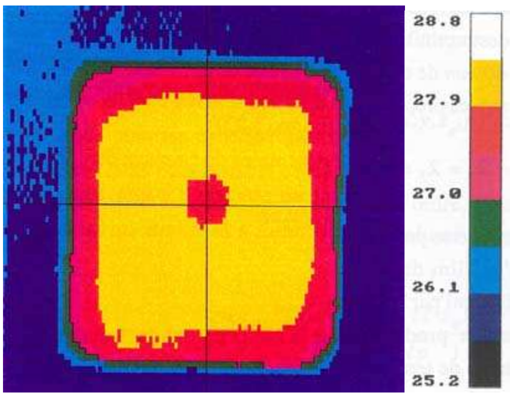

Figure 2. Infrared image at time $t = 1.4\ s$ after the flash heating in a rear face experiment.

Application of Equation 4 with the preceding value of *R*, and $e_1 = 0.5$, can yield a value for the area *(a x b)* once a value has been chosen for the Laplace variable *p*. The optimal value of *p* was found to be equal to *6*. Since our modeling does not take into account the lateral heat losses, the integration domain has been reduced to overcome the edge effects ($L \times \ell$ = 48 mm x 42 mm, instead of 60 mm x

60 mm). The experimental Laplace contrast field (for $p = 6$) was calculated; a profile of this contrast crossing the center of the defect is reported on Figure 3.

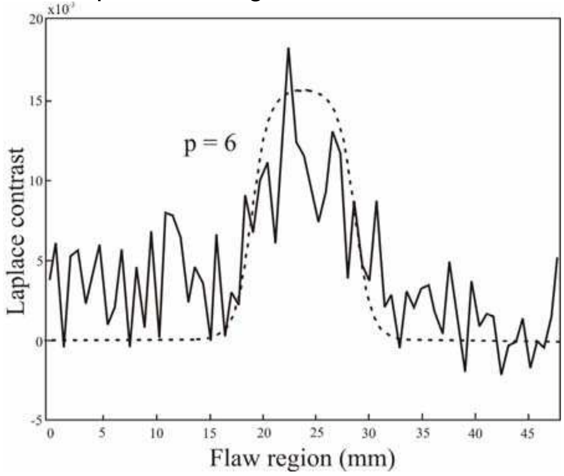

Figure 3. Experimental and theoretical Laplace profiles across the defective region.

The signal is disturbed with a great amount of measurement noise. The corresponding average value of the Laplace contrast is $\Delta\bar{\tau} = -18 \times 10^{-4}$. Since $a = b$, the estimated value of a using Equation 4 is *8.9 mm*, which represents a discrepancy of – 11% with respect to the nominal value. A simulated Laplace contrast distribution calculated with $p = 6$, $e_1 = 0.5$, $a = b = 10$ *mm*, and $R = 0.185$ is obtained using Equation 5 and the inverse Fourier transform; it is plotted in Figure 4. A theoretical Laplace profile above the center of the defective region is derived from the latter figure and reported in Figure 3 together with the experimental profile. The theoretical profile shows that the region affected by the Laplace contrast is approximately 20×20 mm^2, which is not the case for the experiment where noise spreads outside this zone. It should be pointed out however that even with the highly noisy data; the proposed inversion approach has led to an appropriate estimation for the flaw parameters.

5. Conclusion

Unsteady three-dimensional heat diffusion within an anisotropic material containing a limited extent discontinuity has been solved using a perturbation expansion and integral transforms. Parameter estimation procedures based on the solutions obtained via perturbations method and 3D thermal quadrupoles have been implemented. The theoretical results were then validated on an artificial sample submitted to a pulsed thermography test. The flaw was composed by the superimposition of two thin Teflon® films. The estimated lateral extent of the flaw was very close to the nominal value. However, the flaw thermal resistance was more than twice higher than the nominal value. This was explained by air layers trapped between the Teflon® films or between the Teflon® films and the matrix of the composite material.

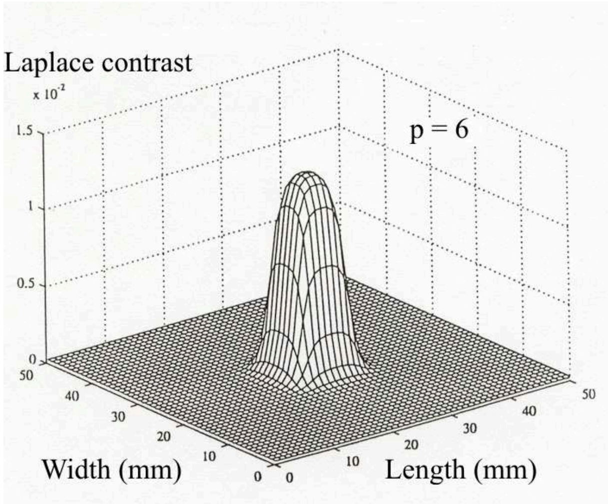

Figure 4. Theoretical Laplace distribution calculated for a depth e_1 = *0.5*, a lateral extent *a x b = 10 mm x 10 mm,* and a thermal resistance *R = 0.185.*

REFERENCES

[1] Maillet (D.), André (S.), Batsale (J.C.), Degiovanni (A.) and Moyne (C.). - *Thermal quadrupoles: solving the heat equation through integral transforms*. Chichester, Whiley, 370 p., 2000.
[2] Aziz (A.) and Na (T.). - *Perturbations Methods in Heat Transfer*. Berlin, Springer-Verlag, 1984.]
[3] Bendada (A.). - *Approximate solutions to three-dimensional unsteady heat conduction through plane flaws within anisotropic media using a perturbation method.* Modeling and Simulation in Material Science and Engineering, vol. 10, 2002, p. 673-684.
[4] Stehfest (H.). - *Remarks on Algorithm 368, Numerical Inversion of Laplace Transforms.* Commun. ACM, vol. 13, 1970, p. 47-49.
[5] Maillet (D.), Batsale (J.C.), Bendada (A.), Degiovanni (A.). – *Integral methods and nondestructive testing through stimulated infrared thermography.* International Journal of Thermal Science, vol. 35, 1996, p. 14-27.